



Printed Pages : 8

EAS - 203

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610

Roll No.

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B. Tech.

(SEM. II) EXAMINATION, 2008-09

MATHEMATICS - II

Time : 3 Hours]

[Total Marks : 100

SECTION - A

Note : Attempt all questions. All questions $2 \times 10 = 20$ carry equal marks.

Fill up the appropriate answers in the space provided :

1 (a) The differential equation

$$\left(\frac{d^3 y}{dx^3} \right)^4 - 6x^2 \left(\frac{dy}{dx} \right)^2 + e^x = \sin xy \text{ is of}$$

_____ order and _____ degree.

(b) A particular solution of the differential equation

$$\frac{dy}{dt} = \frac{1}{2} (y^2 - 1) \text{ that satisfies the initial condition}$$

$$y(0) = 2 \text{ is } \underline{\hspace{2cm}}.$$

(c) The general solution of the differential equation

$$\frac{d^5 y}{dx^5} - \frac{d^3 y}{dx^3} = 0 \text{ is given by } \underline{\hspace{2cm}}.$$



Pick up the Correct Answer from the following :

(d) The Rodrigues formula for Legendre Polynomial

$P_n(x)$ is given by

(i) $P_n(x) = \frac{1}{n! 2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$

(ii) $P_n(x) = \frac{n!}{2^n} \frac{d^n}{dx^n} (x^2 - 1)^{n-1}$

(iii) $P_n(x) = \frac{n!}{2^{n-1}} \frac{d^n}{dx^n} (x^2 - 1)^{n-1}$

(iv) $P_n(x) = \frac{1}{n! 2^n} (x^2 - 1)^n$

(e) The Laplace transform of the function

$f(t) = \begin{cases} 1, & 0 \leq t < 2 \\ -1, & 2 \leq t < 4 \end{cases}$ $f(t+4) = f(t)$ is given as

(i) $\frac{1 - e^{-2s}}{s(1 + e^{-2s})}$

(ii) $\frac{1 + e^{-2s}}{s(1 + e^{-2s})}$

(iii) 0

(iv) $\frac{s+1}{s-1}$

(f) The inverse Laplace transform of $\log \left(\frac{s+1}{s-1} \right)$ is given by

(i) $\frac{2}{t} \cosh t$

(ii) $\frac{2}{t} \sinh t$

(iii) $2t \cos t$

(iv) $2t \sin t$

Indicate **True / False** for the statements made therein -

(g) (i) A function $f(x)$ is even if $f(-x) = -f(x)$ (True / False)

(ii) A function $f(x)$ is odd if $f(-x) = f(x)$. (True / False)

(iii) Most functions are neither even nor odd. (True / False)

(iv) A function $f(x)$ can always be expressed as an arithmetic mean of an even and odd function as

$$f(x) = \frac{1}{2} [f(x) + f(-x)] + \frac{1}{2} [f(x) - f(-x)].$$

(True / False)

(h) (i) With usual symbols, the PDE

$$u_{xx} + u^2 u_{yy} = f(xy)$$

is non-linear in 'u' and is of second order. (True / False)



SECTION - B

Note : Attempt any **three** questions from this section. All questions carry equal marks. **3×10=30**

- 2 (a) Apply the method of variation of parameters to solve the ordinary differential equations
- $$\frac{d^2 y}{dx^2} + y = \tan x, \quad 0 < x < \pi/2.$$
- (b) Show that Bessel's function $J_n(x)$ is an even function when n is even and is odd function when n is odd. Express $J_6(x)$ in terms of $J_0(x)$ and $J_1(x)$.
- (c) Use convolution theorem to find the inverse of the function $\frac{1}{(s^2 + a^2)^2}$.
- (d) Obtain the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)$ in the interval $(0, 2\pi)$ and hence deduce
- $$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$
- (e) A string of Length ' L ' is stretched and fastened to two fixed points. Find the solution of the wave equation $y_{tt} = a^2 y_{xx}$ when initial displacement is
- $$y(x, 0) = f(x) = b \sin\left(\frac{\pi x}{L}\right), \quad \text{where symbols have usual meaning.}$$



- (ii) The small transverse vibrations of a string are governed by one dimensional heat equation

$$y_t = a^2 y_{xx}. \quad (\text{True / False})$$

- (iii) Two dimensional steady state heat flow is given by Laplace's equation $u_t = a^2(u_{xx} + u_{tt})$. (True / False)

- (iv) The PDE of all sphere whose centre lie on z-axis and given by equations $x^2 + y^2 + (z - a)^2 = b^2$, a and b being constants, are governed by $x z_y - y z_x = 0$. (True / False)

- (i) Applying the method of separation of variables techniques, the solution to the P.D.E. $3u_x + 2u_y = 0$

is _____, where $u_x = \frac{\partial u}{\partial x}$, $u_y = \frac{\partial u}{\partial y}$.

Match the column for the items of the Left side to that of right side :

- (j) A second order P.D.E. in the function ' u ' of two independent variables x, y given with usual symbols

$$Au_{xx} + Bu_{xy} + Cu_{yy} + F(u) = 0, \text{ then}$$

- | | |
|---------------------|---------------------|
| (i) Hyperbolic | (a) $B^2 - 4AC = 0$ |
| (ii) Parabolic | (b) $B^2 - 4AC < 0$ |
| (iii) Elliptic | (c) $B^2 - 4AC > 0$ |
| (iv) Not classified | (d) $A = B = C = 0$ |



SECTION - C

Note :

Attempt all questions from this section, $[5 \times 2] \times 5 = 50$ selecting any **two** parts from each question.

All questions carry **equal** marks.

- 3 (a) The equations of motion of a particle are given by $\frac{dx}{dt} + wy = 0$, $\frac{dy}{dt} - wx = 0$. Find the path of the particle and show that it is a circle.

- (b) Integrate the differential equation

$$\frac{d^2x}{dt^2} + 2x \frac{dx}{dt} + w^2x = a \cos pt \text{ and give the}$$

physical interpretation of the complete solution. Also deduce that the solution takes the form

$$x = \frac{a \cos(pt - \theta)}{\sqrt{(w^2 - p^2)^2 + 4k^2 p^2}} \left(\tan \theta = \frac{2kp}{w^2 - p^2} \right) \text{ As } t \rightarrow \infty$$

- (c) Find the complete solution of the differential

$$\text{equation } \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = x e^x \cos x.$$

- 4 (a) For the Bessel's function, prove that

$$J_{\frac{1}{2}}(x) = \left(\sqrt{\frac{2}{\pi x}} \right) \cdot \sin x.$$

- (b) Estimate the recurrence relation for Legendre's polynomials -

$$x P'_n(x) = n P_n(x) + P'_{n-1}(x)$$

- (c) Express the polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomials.

- 5 (a) Using Laplace transformation, show that

$$\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}.$$

- (b) A particle moves in a line so that its displacement x from a fixed point 0 at any time t , is given by

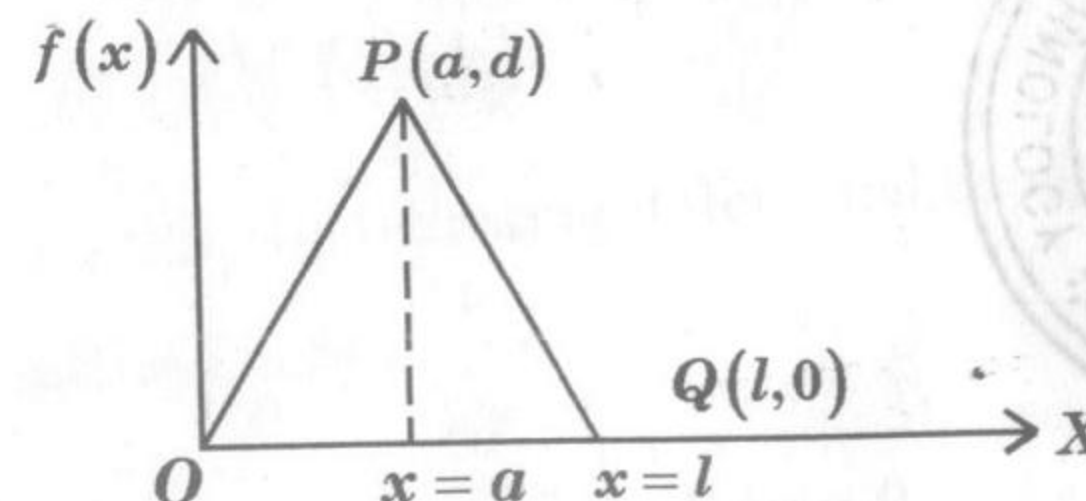
$$\frac{d^2x}{dt^2} + 4 \frac{dx}{dt} + 5x = 80 \sin 5t. \text{ Using Laplace}$$

transform, find its displacement at any time t if initially particle is at rest at $x = 0$.

- (c) Find the Laplace transform of the function

$$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$

- 6 (a) Find the half period sine series for $f(x)$ given in the range $(0, l)$ by the graph OPQ as shown in figure.



(b) Find the Fourier series expansion for

$$f(x) = x + \frac{x^2}{4}, \quad -\pi \leq x \leq \pi.$$

(c) Solve the P.D.E.

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$$

(a) Find the temperature in a bar of length 2 whose end are kept at zero and lateral surface is insulated if

the initial temperature is $\sin \frac{nx}{2} + 3 \sin 5\pi \frac{x}{2}$.

(b) Apply the method of separation of variables to

solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

(c) Solve the P.D.E. by separation of variables method,

$$u_{xx} = u_y + 2u, \quad u(0, y) = 0,$$

$$\frac{\partial}{\partial x} u(0, y) = 1 + e^{-3y}$$

